Risk Parity: The Democratization of Risk in Asset Allocation

Francesco A. Fabozzi, Joseph Simonian, and Frank J. Fabozzi

Francesco A. Fabozzi
is a doctoral student in data science at the Stevens Institute of Technology in Hoboken, NJ.
ffabozzi@stevens.edu

Joseph Simonian
is the founder and CIO of Autonomous Investment Technologies LLC in Newton, MA.
jsimonian@autoinvestec.com

Frank J. Fabozzi
is professor of finance at EDHEC Business School in Nice, France.
frank.fabozzi@edhec.edu

KEY FINDINGS

- Unlike mean–variance optimization, a risk parity strategy allocates across asset classes such that each asset class contributes equally to portfolio risk, regardless of its expected returns.
- In practice, there are variants of the risk parity strategy due to choices made by the portfolio manager, such as selection of the asset classes, risk measure, targeted volatility, degree of leverage, asset selection using active or passive approach, and tactical risk allocation strategy.
- The performance of the risk parity strategy has varied, with critics of the strategy identifying theoretical and practical implementation issues.

ABSTRACT

The risk parity investment model for asset allocation offers an alternative to the mean–variance framework. The fundamental idea is that the allocation to different asset classes should not be based on an optimization that targets a specific return with a minimal level of risk but, rather, should generate a portfolio in which the contribution to portfolio risk of each asset class is equal, regardless of its expected returns. In this article, the authors explain the fundamentals of the risk parity investment model and the variants in risk parity strategies due to the selection of the asset classes to be included in the portfolio, the choice of the risk metric, the portfolio risk target, how to obtain leverage, associated leverage, whether the selection of the specific investments within an asset class is made using an active or passive approach, and the tactical risk allocation strategy. In addition to describing the practical aspects of implementing risk parity strategies, the authors identify the various shortcomings of the model and some extensions of the basic risk parity model that attempt to address some of the issues identified by the model's critics.

TOPICS

Portfolio theory, portfolio construction, risk management, performance measurement*

Until the late 1990s, traditional asset allocation approaches primarily focused on exposure to equities. An allocation of 60% equity and 40% bonds has been labeled *traditional asset allocation*. The 60/40 model has been used as a benchmark in comparing the performance of most multi-asset strategies. The allocation to bonds is the means by which a fixed asset allocation is used to control risk. The view is that the allocation to bonds reduces the risk exposure of a traditional asset

*All articles are now categorized by topics and subtopics. **View at PM-Research.com**.

allocation strategy by serving as a counterbalance to equity.[1] The mean–variance framework introduced by Markowitz (1952) builds on this notion by providing a framework that formalizes the concept of diversification by stipulating that the correlation between asset classes is the key to balancing risk in portfolios.

An alternative to the mean–variance framework is the risk parity strategy. With risk parity, the idea is not to allocate to different asset classes based on capital allocation using an optimization that targets a specific return with a minimal level of risk but, rather, to generate a portfolio in which the contribution to portfolio risk of each asset class is equal, regardless of their expected returns. The primary advantage of a risk parity strategy for asset allocation is that the need to forecast or posit expected returns is dispensed with a more egalitarian dispersion of risk among a portfolio's constituent asset classes. This is a significant potential advantage over the traditional 60/40 model in which, historically, as much as 90% of a portfolio's volatility is due to its equity exposure, and its performance is mostly determined by equity returns. According to Maillard, Roncalli, and Teïletche (2010), an unlevered risk parity portfolio creates a portfolio similar to that provided by the minimum variance portfolio, save for the fact that each asset class contributes equally to the portfolio's risk.

Risk parity strategies have become popular products over the years. Estimates by the firm Neuberger Berman suggest that there is about $120 billion of assets under management following risk parity strategies, or about $400 billion to $500 billion with leverage.[2] The popularity of this strategy has resulted in the creation of a risk parity index series by S&P as a benchmark for equal-weighted parity strategies (see Liu, Brzenk, and Cheng 2020).

In this article, we describe the basic workings of the risk parity portfolio allocation model. Practical aspects of implementing risk parity strategies are also discussed, as are various shortcomings of the model pertaining to empirical performance and theoretical and implementation challenges. We also mention extensions of the basic risk parity model that attempt to address some of the foregoing issues.

A BRIEF HISTORY OF RISK PARITY PRODUCTS

The history of risk parity does not trace back to the hallowed halls of any esteemed academic institution. Rather, the foundations of the risk parity strategy were set forth in publications by Ray Dalio and his portfolio management team at Bridgewater Associates. Bridgewater developed risk parity as the core investment strategy for the firm's All Weather Fund that started in 1996.

The underlying economic thesis of the All Weather Fund is that different asset classes possess excess return over the risk-free rate, and they react differently to two macroeconomic risks of the business cycle: growth and inflation. The value of an asset is equal to the present value of its expected future cash flows, which are affected by changes in economic conditions. For example, expected economic growth and inflation affect corporate future cash flows. The level of interest rates affects the discount rate or the yield that is used to discount future cash flows. As these expectations about future economic conditions change, the value of assets and asset classes changes.

Given these fundamental asset pricing assumptions, Bob Prince, co-CIO of Bridgewater Associates, argued that the two main drivers of the valuation of asset classes are (1) the accrual of and changes in the risk premium and (2) unanticipated shifts in the economic environment (see Prince 2011). Given these two main drivers of

[1] Hence the use of the term *balanced fund* to describe many 60/40 strategies.

[2] As reported by Davies and Keidan (2020).

valuation, the objective in establishing a strategic asset allocation that provides diversification is, in the words of Prince, to "collect the risk premium as consistently as possible, by minimizing risk due to unexpected changes in the economic environment."

The two major risks that Bridgewater uses in managing its All Weather Fund are growth and inflation. Accordingly, the four risk scenarios used to capture the economic scenarios are

- growth rising relative to expectations
- growth falling relative to expectations
- inflation rising relative to expectations
- inflation falling relative to expectations

The aforementioned four risk scenarios can be viewed as subportfolios in which an investor can allocate funds. The investor is assumed to have no view that financial markets have any tendency to under- or overdiscount the two economic drivers (growth and inflation) in the future. Thus, portfolio construction in the risk parity strategy used by the All Weather Fund allows 25% of risk to be allocated to each of these categories.

How does this approach to portfolio construction generate a diversified portfolio in different economic environments? Consider the case in which bonds underperform in an economic environment due to higher than expected inflation. This would be offset by an asset class such as commodities that outperforms because its sensitivity to inflation is the opposite to that of bonds. The intent of this form of diversification is to produce a more stable portfolio return.

The Bridgewater All Weather approach is rather qualitative compared to most risk parity providers. Although it falls into the risk parity category, it is an exception because it was developed before the quantitative risk parity concept was formally proposed. Others select assets with equity risk, interest rate risk, and inflation-related risk; assign a balanced risk allocation to select assets; build a covariance matrix; solve for an unlevered risk parity portfolio; and then apply appropriate leverage based on targeted risk, as described later in this article.

At the time Bridgewater went to market, there was no dissatisfaction with the 60/40 model given the bull market of the 1990s and the benefits of an allocation strategy that was exposed to equities. In March 2000, at the advent of the Tech Wreck, the risks associated with overexposure to equities became apparent. The investment community then turned its attention to controlling equity risk and portfolio volatility. Industry research at the time suggested potentially substantial benefits to pursuing a risk parity strategy. However, it was not until 2005 that the term *risk parity* was coined by Edward Qian of PanAgora Asset Management in a white paper (Qian 2005).[3] Various risk parity strategies were subsequently marketed by several major asset management firms.

ESSENTIAL ELEMENTS OF A RISK PARITY STRATEGY

The risk parity strategy does not seek to identify the optimal portfolio from mean–variance analysis for a given level of expected return but instead to construct a portfolio such that the risk contribution to the portfolio of each asset class is roughly the same. Mean–variance analysis often provides an efficient portfolio that is heavily concentrated in a few risk assets such as equities and commodities. Therefore, the efficient portfolio identified has considerable exposure to volatility spikes

[3] For a history of risk parity based on entries in *Wikipedia*, see McDaniel (2012). Qian has gone on to publish several important papers on risk parity (Qian 2011, 2013) and a book (Qian 2016).

because the latter asset classes expose investors to more near-term downside than other asset classes such as bonds. Aside from the mean–variance portfolio, many industry benchmarks such as the 60/40 portfolio also heavily concentrate their risk in one asset class—equities. Although 60/40 portfolios superficially divide capital between stocks and bonds almost evenly, their risk is in fact driven almost entirely by equities. Thus, from the standpoint of theory and practice, the risk parity strategy can be viewed as yet another framework seeking to bring genuine diversification to portfolio construction.

Formal descriptions[4] of the risk parity strategy typically begin by defining the volatility contribution of risk for each asset class $\sigma_i(w) = w_i \times \partial_{w_i}\sigma(w)$ where portfolio volatility is $\sigma(w) = \sqrt{w^T \Sigma w}$. Given this, it is plain that $\sigma(w) = \Sigma_{i=1}^{n}\sigma_i(w)$ and the vector of marginal contributions $\partial_{w_i}\sigma(w)$ is derived as follows:

$$mc(w) = \frac{\Sigma w}{\sqrt{w^T \Sigma w}} \tag{1}$$

Accordingly, for N assets, the objective function to determine the risk parity portfolio weights is

$$\arg\min_{w} \sum_{i=1}^{N} \left[\frac{\sqrt{w^T \Sigma w}}{N} - w_i \times mc(w)_i \right]^2 \tag{2}$$

Unlike mean–variance analysis, which is a normative economic theory, risk parity is more diversification focused and empirically driven based on historical evidence about how assets are priced. The key is that portfolios should be structured such that there is an equal marginal risk contribution from each asset class. From an implementation standpoint, perhaps the most critical aspect of a risk parity strategy is its use of leverage, specifically, the levering of the entire portfolio to enhance returns. For most investors or plan sponsors, the expected return of an unlevered risk parity strategy is insufficient compared to their required return hurdles. The application of leverage to risk parity helps align the portfolio with an investor's return objectives without compromising on stability objectives.

The implementation of a risk parity strategy involves several decisions that must be made by an asset manager: (1) selection of the risk metric and the target level of risk, (2) selection of the asset classes to be included in the portfolio, (3) how to estimate the risk contribution of each selected asset class, (4) how to construct an equally risk-weighted portfolio with no leverage (i.e., an unlevered risk parity portfolio), (5) how to create leverage to construct a levered risk parity portfolio, and (6) how to make tactical expressions away from the strategic risk allocation of the portfolio. Funds pursuing risk parity strategies can be distinguished based on the foregoing six decisions.[5]

What is the appropriate risk measure in a risk parity strategy? Although most risk parity managers use the standard deviation of asset class returns as the risk metric, other measures can be used, such as downside deviation, maximum drawdown, or conditional value-at-risk. Despite the well-known problems with using standard deviation as a measure of risk, it appears to be the risk measure of choice in most risk parity products because it is convenient for decomposing a portfolio's volatility into volatility contributions from each of the candidate asset classes. The target level of

[4] Maillard, Roncalli, and Teïletche (2010) provided a detailed discussion of some of the mathematical properties of risk parity portfolios.

[5] Because of these decisions in pursuing a risk parity strategy, there is a series of S&P risk parity benchmarks based on the target volatility. The four target volatility levels are 8%, 10%, 12%, and 15%.

risk and leverage can be based on a variety of considerations, with client suitability being the primary one.[6]

The universe of potential asset classes to include in a risk parity portfolio has grown over the years. The universe of asset classes today sometimes includes alternative asset classes, including those exhibiting a relatively high degree of illiquidity. The inclusion of extended asset classes will typically necessitate augmenting the basic risk parity framework to account for additional types of risk, including illiquidity and event risk.

Leverage can be applied to a risk parity portfolio either explicitly or implicitly. Explicit leverage is created through borrowing funds from a counterparty (e.g., prime broker), and fund assets are used to collateralize the loan. Prime broker financing and repurchase agreements are examples of explicit leverage. Implicit leverage is created through the use of derivative instruments in which the notional exposure is greater than the margin required to collateralize the position. Futures, forwards, swaps, and options are all examples of implicit leverage. The choice of leverage has corresponding impacts on liquidity, counterparty risk, and trading costs.

Various empirical studies on asset pricing have been useful in refining investors' implementation of risk parity strategies, in particular studies dealing with what is referred to as the *low-volatility anomaly*. The latter have shown evidence that relatively low-volatility asset classes exhibit a better return per unit of risk than high-volatility asset classes.[7] This was also found to be true within asset classes.[8] Risk parity therefore involves constructing an unlevered portfolio that may have a low expected return because of a relative overweighting to bonds that then uses leverage to increase exposure to bonds, which have a better return per unit of risk relative to other asset classes. At one time, the bond sector of choice that offered the most desirable investment attributes for risk parity was fixed-rate US Treasury bonds. However, in the current low-interest-rate environment, some portfolio managers have gravitated toward US Treasury inflation protection securities or risky sovereign market bonds to enhance returns.[9]

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[6] Creating a levered portfolio is also part of the selection of an optimal portfolio using mean–variance analysis to achieve a target return. In this case, what is being levered is the market portfolio.

[7] Two reasons have been offered as an explanation for the low-volatility anomaly. The first is based on the questionable assumption in the capital asset pricing model that financial markets are frictionless. If financial markets were indeed frictionless, then some strategies would bring the risk–return relationship in line so that no stock or asset class would offer a superior return per unit of risk. The mechanisms available to market participants to bring the risk–return relationship for all assets and all asset classes in line are short selling and leveraging. The costs and risks associated with these two mechanisms make investors reluctant to use them. Consider the impact of the aversion to leveraging on the pricing of high-risk and low-risk assets resulting from the demand for these assets. For the pricing of low-risk assets, the reluctance of market participants to leverage low-risk assets reduces their demand. This results in low-risk assets being undervalued and therefore offering a high return relative to risk. For high-risk assets, market participants, in seeking to increase expected return to meet a target return, create increased demand and therefore cause the assets to be overvalued. As a result, high-risk assets offer a lower return relative to risk when compared to low-risk assets. Almost 50 years ago, Black (1972) provided support for this view. He hypothesized that, because investors did not want to lever their portfolios (i.e., they were leverage averse), assets that had lower risk would outperform on a risk-adjusted basis compared to those with higher risk. The second reason offered to explain the low-volatility anomaly is that some investors prefer lottery-type investments. Because high-risk assets offer a lottery-type payoff, the demand for such assets reduces their return per unit of risk compared to low-risk assets.

[8] See Frazzini and Pedersen (2013).

[9] In an article by Davies and Keidan (2020), Bridgewater's CIO, Bob Prince, for example, indicated that the firm has recently reduced its exposure to Treasuries and increased its exposure to inflation-linked bonds and to China. In that same article, a Neuberger Berman senior portfolio manager stated that it was rotating into Treasury inflation protection bonds. Prince also argued that the 60/40 model was likely disadvantaged even more in the current market environment relative to a levered risk parity strategy as a result of both a lower return for bonds and higher equity downside risk. A separate study by Belton

Although there is a potential return benefit from the latter tactic, it also burdens risk parity portfolios with increasing liquidity risk.

In selecting the specific assets for each asset class, a risk parity strategy can be done through a passive strategy using a low-cost collective investment vehicle such as an exchange-traded fund[10] or an active strategy such as one making use of a factor model. It is also possible to focus the equity portion of a portfolio on a specific investment style such as value or growth, a specific region, or a specific market cap. Finally, we note that over time the level of risk of the unlevered portfolio will change, sometimes significantly. Thus, to maintain the same target level of risk, the amount of leverage must be periodically adjusted. This can be a systematic process or an active decision to derisk or rerisk the entire portfolio.

CHALLENGES WITH THE RISK PARITY STRATEGY

With the popularity of risk parity strategies increasing, some investors and asset owners have raised issues about various aspects of this investment model. The criticisms have addressed empirical performance, theoretical issues, and problems with implementation.

We begin with empirical studies of performance.[11] Because there are several generic risk parity strategies, empirically assessing their performance has been difficult. As with all investment models, in some periods they have worked well relative to competitor models and at other times worse.[12] For example, a 2010 research report by Dimensional Fund Advisors found that risk parity failed to outperform the 60/40 model (Marlena 2011). Specifically, the study used 81 years of returns from 19 countries to conduct an out-of-sample test of whether risk parity delivers superior risk-return tradeoffs. The results show that previously documented risk parity benefits are sample specific. Over the last 81 years, risk parity portfolios do not have higher Sharpe ratios than 60/40 balanced portfolios.

The study goes on to explain why

> Out-of-sample results show that the touted benefits of risk parity only appear in the last thirty years during a period of falling inflation and interest rates. Because bonds did unexpectedly well over this period risk parity portfolios also benefit due to their heavy bond allocations. Unsurprisingly, risk parity does poorly from 1956 to 1980, a period of rising inflation. From 1930 to 1955, a period of volatile but non-trending inflation, risk parity yields Sharpe ratios that are similar to traditional 60/40 portfolios.

A 2010 study by Callan Associates (2010) examined generic risk parity strategies for the 20-year period from 1990 to 2010. Consistent with the findings of the Dimensional Fund study, the study found that levered risk parity would have underperformed a typical institutional portfolio in the 1990s. Thus, all the outperformance was generated from 2000 to 2010. The study concluded that

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and Qian (2020) of PanAgora Asset Management determined that using low-yielding sovereign bonds would not have hindered the performance of a risk parity strategy in the current market environment.

[10] For the S&P Risk Parity Index Series, futures are used to represent asset classes. Because futures are leveraged instruments, a levered portfolio can be created.

[11] For more recent and realistic performance of the risk parity strategy, see the S&P risk parity index and HFR risk parity index.

[12] For example, in the case of smart beta strategies, a study by the Schwab Center for Financial Research by Davidow (2018) found that no single strategy dominated over the period 2008 to 2017.

> Given the underperformance in the 1990s and the practical difficulties of implementing a levered policy portfolio, it is unlikely that the levered Risk Parity approach would have survived at any major institution during that time in order to deliver on its promise of risk control during the downturns of the last decade.

From a theoretical standpoint, a major advantage of risk parity for some investors is that it does not require asset class return forecasts. However, some strategists argue that ignoring returns is a major drawback of the model. The argument is that the role of an asset manager is to maximize return and not to minimize risk. Consequently, despite the difficulty in projecting asset class returns, some strategists have argued that the weighting for each asset class should be based on its Sharpe ratio, which involves estimating expected returns. This approach, suggested by UBS Global Research strategists, is referred to as the *Sharpe parity* approach.[13] As cited by Razza (2014), in the view of UBS, "the Sharpe ratio weighted strategy has lower risk relative to risk parity due to its broader asset class exposure. On absolute performance terms, the Sharpe ratio weighted strategy outperforms other strategies including risk parity." Others have suggested weighting asset classes using other risk–return measures, such as the Sortino ratio.

A second theoretical issue with the risk parity model is related to its use of a covariance matrix. This raises the same basic dimensionality problem that is faced by mean–variance optimization, namely that reliable variance and covariance forecasts for numerous assets would presumably still need to be made to implement the model. Even if only historical data are used, there is still the potential challenge that some assets may not have a long enough history to model them with confidence. Thus, even if the risk-only nature of risk parity models simplifies the modeling challenge, it does not eliminate it. One remedy to the dimensionality issue is to use a factor risk parity approach, such as that described by Roncalli and Weisang (2016).

A third related theoretical issue, and one also connected to the input covariance matrix, is that it is susceptible to the instability of portfolio outcomes that plagues any optimization exercise that relies on quadratic programming techniques, such as the critical line algorithm (introduced by Markowitz 1956). It is often the case that small changes in optimization inputs result in the production of significantly different portfolios. This is especially the case in the basic mean–variance framework. By dispensing with expected returns, risk parity approaches remedy this issue to some extent, but they do not eliminate it. This is so because quadratic programming methods necessitate the inversion of a positive-definite covariance matrix. One approach, introduced by Lopez de Prado (2016), attempts to remedy the instability problem by means of hierarchical clustering. According to some practitioners, this is not a typical concern for risk parity investors if they use a slow decay rate to estimate the covariance matrix. Moreover, the risk parity process does not use an optimizer.

Finally, we note that as in the case of the mean–variance framework, the risk parity model also views risk only as variance, ignoring skewness and kurtosis. This shortcoming potentially places risk parity portfolios in a vulnerable position in the event of large, unexpected market dislocations (black swan events). There are two approaches to rectifying the incomplete picture of risk in models like the risk parity model. The first

[13] This was cited by Razza (2014) regarding a UBS white paper by Stephane Deo and Ramin Nakisa ("Weight Watcher—Much Better Than Risk Parity: Sharpe Parity"). The cited paper has been removed from the Internet. See also the following publication by the *Chief Investment Officer* website posting "Sharpe Parity: The New Risk Parity" available at https://www.ai-cio.com/news/sharpe-parity-the-new-risk-parity/.

is to explicitly incorporate skewness or kurtosis.[14] The second is to use a so-called robust approach to portfolio selection, which calibrates input parameters to worst-case values based on the model builder's confidence in them.[15]

In terms of practical challenges, there are three primary possible impediments to the successful implementation of risk parity strategies. The first involves their use of leverage. Risk parity strategies typically attempt to use leverage to increase their expected returns. Because leverage involves borrowing, risk parity strategies are susceptible to sudden or significant jumps in borrowing costs. (This is only true for explicit leverage.) A second implementation challenge is that in some market environments, such as the one we find ourselves in now, most reasonable levels of leverage would not be able to provide a level of expected returns that would justify the additional risk of levering the portfolio up. This is due primarily to the ultra-low yields we observe in the bond market. However, low yields are not a challenge unique to risk parity; they are a greater challenge for a 60/40 strategy that does not incorporate any leverage.

A third implementation challenge is that asset correlations, namely those between stocks and bonds, may not behave as expected. This was the case in the second quarter of 2013 when the US Federal Reserve announced that it was reducing quantitative easing. Rather than moving in different directions, stocks and bonds moved in unison during that time. Although any passive multi-asset portfolio's performance would have suffered during that period, risk parity strategies (especially passive ones) did not possess any special immunity to that particular market scenario. On the other hand, an active risk parity strategy might decrease portfolio risk by cutting leverage. This decision could reduce portfolio drawdown but later hurt portfolio performance drastically when markets recover quickly, as they did in the second quarter of 2020.

CONCLUSION

Although initially presented as an alternative to traditional 60/40-type allocations, risk parity strategies are by now a mainstream asset allocation approach, used by institutional and individual investors alike. The investment motivation for risk parity is no different than that of other core portfolio selection frameworks such as mean–variance optimization: diversification. However, in contrast to mean–variance optimization, risk parity eschews expected return as part of its objective function, instead focusing solely on the equal apportionment of risk among the asset classes in a portfolio. Risk parity portfolios are superior to mean–variance-optimized portfolios in terms of their general avoidance of overly concentrated corner portfolios, as well as their tendency to produce relatively more stable returns through time. That said, risk parity portfolios may underperform more traditional asset allocations given that expected returns play no formal role in generating them. Furthermore, although risk parity portfolios produce more stable outcomes relative to many rival frameworks, they do not completely eliminate the risk of model misspecification, asset selection, and active or tactical decisions. Although risk parity is not the appropriate solution

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[14] See Bruder, Kostyuchyk, and Roncalli (2016) for a risk parity approach that accounts for skewness. We note that any analytical portfolio selection framework that takes into account skewness and kurtosis must also account for co-skewness and co-kurtosis, thus adding further complexity to the modeling process.

[15] Model misspecification is a concern for every quantitative asset manager. As a result, for some time there has been an effort to extend the static max-min expected utility theory framework of Gilboa and Schmeidler (1989) to a dynamic environment in various areas of finance, including portfolio construction. Gilboa and Schmeidler's framework provides an elegant way of modeling decision making that incorporates investors' aversion to model uncertainty and ambiguity in the sense described by Ellsberg (1961).

for every type of investor, the strategy has nevertheless taken its place as a popular and logical approach for many investors concerned about risk control and the stability of investment outcomes.

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